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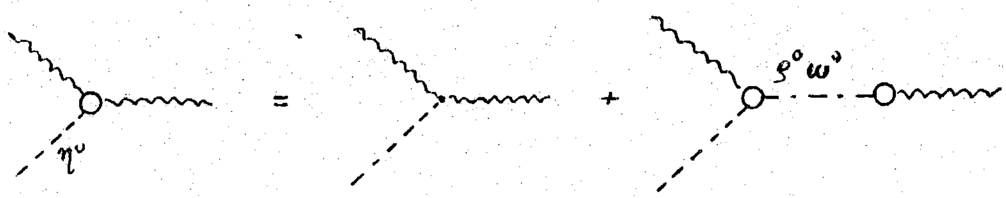
E. Celeghini[(x)] and R. Gatto: THE DECAY MODES $\eta^o \to \gamma + e^+ + e^-$ AND $\eta^o \to \gamma + \mu^+ + \mu^-$.

In this note we shortly report on the results of a calculation of the spectra and probabilities of $\eta^o \to \gamma + e^+ + e^-$ and $\eta^o \to \gamma + e^+ + e^-$. We assume for η^o the quantum numbers $J = 0$, $P = -1$, $I = 0$, $C = G = +1$. We approximate the $\eta^o \to 2\gamma$ amplitude (with one photon off-mass-shell) with a subtraction term and the pole terms due to ρ^o and ω^o:

(1) 

We call m the invariant mass of the emitted two-lepton system

(2) $$m = \left[(E_+ + E_-)^2 - (p_+ + p_-)^2 \right]^{1/2}$$

where $E_\pm$ and $p_\pm$ are the energies and momenta of the positive (negative) lepton. The possible values of m are between $2m_\ell$ (where m_ℓ is the lepton mass) and m_η (η - mass $\simeq$ 550 MeV). The number of events with m between m and m + dm is given by:

(x) - Istituto di Fisica dell'Università di Firenze.

$$N(m)\,dm = \left[\tau(2\gamma)\right]^{-1} \frac{4\alpha}{3\pi} \frac{1}{m} \left[1 - \left(\frac{m}{m_\eta}\right)^2\right]^3 \left[1+2\left(\frac{m_\ell}{m}\right)^2\right] \times \tag{3}$$

$$\times \left[1 - \left(\frac{2m_\ell}{m}\right)^2\right]^{1/2} \left[v \frac{m_V^2}{m_V^2 - m^2} + (1-v)\right]^2 dm$$

where v is a parameter depending on the relative values of the residui of the poles and the subtraction constant. In (3) $\tau(2\gamma)$ is the partial lifetime for $\eta^0 \to 2\gamma$. The mass m_V is some average of the ρ^0 and ω^0 mass. The value v=0 corresponds to keeping only the subtraction term in the expansion (1) (constant form factor). The value v = 1 corresponds to keeping only the vector meson poles neglecting the subtraction constant. There exists an experiment on the π^0 form-factor that gives the value of the derivative at the origin of the π^0 form-factor with respect to the squared four-momentum of the off-mass-shell photon(1). If we assume that η^0 is the eighth member of the unitary symmetry octet containing π and K we can tentatively make use of unitary symmetry to obtain a value for v from the quantity measured in the π^0 experiment. We get v = -7 ± 5. Needless to say, such an extrapolation of unitary symmetry arguments to a low energy region where mass differences are quite important may be completely illusory. In fig. 1 we have reported the entire spectrum of $\eta^0 \to \gamma + \mu^+ + \mu^-$ and the high energy tail of $\eta^0 \to \gamma + e^+ + e^-$ in arbitrary units, for values of v = 0, 1 and -7. The spectra have the right relative normalizations, i.e., apart from the common factor $(4\alpha/3\pi)(\tau(2\gamma))^{-1}$, N(m)dm gives, for each case, directly the number of events with m between m and m + dm.

Of course, for $\eta^0 \to \gamma + e^+ + e^-$ the spectrum receives its biggest contribution from smaller values of m down to $2m_e$, as given by (3). The branching ratios ρ_e and ρ_μ for $\eta^0 \to \gamma + e^+ + e^-$ relative to $\eta^0 \to 2\gamma$ can be obtained by integrating (3). For v = 0 one has

$$\rho_\mu = \frac{2\alpha}{3\pi} \left\{ \left(-\frac{7}{2} + 13r^2 + 4r^4\right)(1 - 4r^2)^{1/2} + \right. \tag{4}$$

$$\left. + 2(1 - 18r^4 + 8r^6)\,\lg \frac{1 + (1-4r^2)^{1/2}}{2r} \right\}$$

with $r = m_\mu/m_\eta$. Putting r = 0 one obtains

$$\rho_e = \frac{2\alpha}{3\pi} \left[\log\left(\frac{m}{m_e}\right)^2 - \frac{7}{2}\right] ,$$

(1) - N.P. Samios - Phys. Rev. 121, 275 (1961).

which is the well-know Dalitz formula.

By numerical integration, with m_V = 750 MeV, we find

(5) $$\rho_e = (16.2 + 0.47\, v + 0.035\, v^2) \times 10^{-3}$$

(5') $$\rho_\mu = (55.8 + 21.9\, v + 2.74\, v^2) \times 10^{-5}$$

From (5) and (5') we see that ρ_e is much less sensitive to v than ρ_μ and, taking the model literally, we expect, independently of v, $\rho_e > 14.6 \times 10^{-3}$ and $\rho_\mu > 12.1 \times 10^{-5}$. With v = 0 $\rho_e = 16.2 \times 10^{-3}$ and $\rho_\mu = 55.8 \times 10^{-5}$. With v = 1 $\rho_e = 16.7 \times 10^{-3}$ and $\rho_\mu = 80.4 \times 10^{-5}$. With v = - 7 (unitary symmetry extrapolation) $\rho_e = 14.6 \times 10^{-3}$ and $\rho_\mu = 36.9 \times 10^{-5}$. Note that ρ_e alone would determine v (or better two possible values for v) and then one would be able to predict ρ_μ and the shapes of the spectra. A reaction such as $\gamma + p \rightarrow \eta^o + p$ followed by $\eta^o \rightarrow \mu^+ + \mu^- + \gamma$ simulates $\gamma + p \rightarrow \mu^+ + \mu^- + \gamma + p$. With a cross-section $\sim 10^{-30}$ cm^2 for $\gamma + p \rightarrow \eta^o + p$ the apparent cross - section for $\gamma + p \rightarrow \mu^+ + \mu^- + \gamma + p$ would then be at least $\sim 10^{-34}$ cm^2.

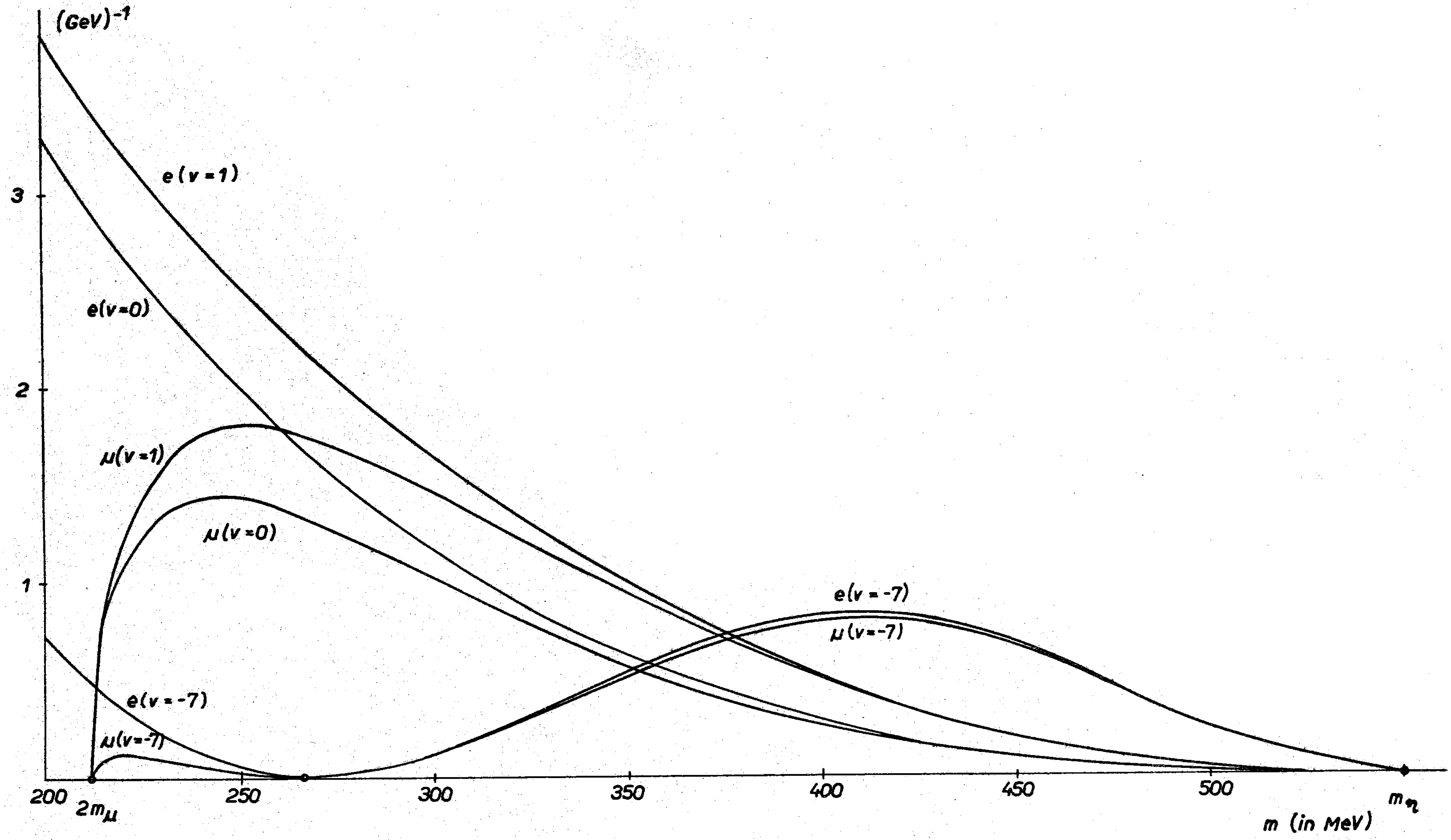


FIG. 1 - Graphs of $\frac{3\pi}{4\alpha}\tau(2\gamma)\,N(m)$, where N(m)dm is the number of events with m (invariant lepton mass) between m and m+dm, for $\eta^0 \rightarrow \gamma + e^+ + e^-$ and $\eta^0 \rightarrow \gamma + \mu^+ + \mu^-$ with different values of the parameter v.